Abstract—In this paper, the singularity of closed structures when using time-domain magnetic field integral equation is analysed. This type of formulation is solely applicable to closed structures and it is tried to show the effect of the singularities existing at the internal resonant frequencies of the closed structure. The results of a closed sphere are compared with analytical solution to demonstrate the minor effect of such singularities. In order to avoid the late-time instabilities, the implicit method of moments is used to solve the resultant integral equations.

I. INTRODUCTION

In EMC problem investigations and analysis, where the transient electromagnetic pulses (EMP) are important sources of interference, frequency-domain methods are not suitable. For such problems, time-domain methods are more efficient and widely used [1]. The electric and magnetic field integral equations, (EFIE) and (MFIE) respectively, are widely used in the scattering problems. Solution of Maxwell’s equations and applying the appropriate boundary conditions results the EFIE and MFIE formulations [2]. Both of these formulations are singular at the frequencies corresponding to the internal resonance frequencies of the considered closed structures. Though the surface currents resulted by the solution of the EFIE and MFIE equations are not unique at such frequencies, the scattered fields of the EFIE solution does not suffer from the singularity. To avoid the singularity a combined Field solution is proposed [3]. The solution of MFIE equation is simpler than the EFIE, furthermore, in problems that dielectric objects are analysed, the MFIE formulation may be applied. In this paper it is tried to show the effects of the described singularity on the transient results of a perfect electric conducting (PEC) sphere. An implicit Method of Moments (MoM) in time domain is implemented to solve the magnetic field integral equation.

II. FORMULATION OF THE PROBLEM

The magnetic field boundary condition on the surface of an ideal conducting closed body $S$ is given by

$$\vec{n} \times \left[ H^i (\vec{r}, t) + H^S (\vec{r}, t) \right] = \vec{J} (\vec{r}, t) \quad \vec{r} \in S$$

(1)

where $\vec{n}$ is the normal vector pointing outward the surface $S$, $H^i$ is the incident magnetic field and $H^S$ is the scattered magnetic field radiated by surface electric currents, which may be written in terms of magnetic vector potential functions

$$H^S (\vec{r}, t) = \frac{i}{\mu} \nabla \times \vec{A} (\vec{r}, t).$$

(2)

In this relation $\mu$ is the permeability of the scattering medium and $\vec{A}$ is magnetic vector potential given by

$$\vec{A} (\vec{r}, t) = \frac{\mu}{4\pi} \int_{S'} \frac{\vec{J} (\vec{r'}, t)}{R} dS'.$$

(3)

where $\tau = t - R/c$, $R = |\vec{R}|$ and $\vec{R} = \vec{r} - \vec{r'}$. The two parameters $\vec{r}$ and $\vec{r'}$ represent vectors from the origin to the observation point and source point respectively. The velocity of light is designated by $c$.

Utilizing the magnetic vector potential defined by (3) in (2) results to

$$H^S (\vec{r}, t) = \frac{i}{4\pi} \nabla \times \int_{S'} \frac{\vec{J} (\vec{r'}, t)}{R} dS'.$$

(4)

In order to remove the singular point $R = 0$ from the surface $S$, Eq. (4) may be rewritten [1] as

$$H^S (\vec{r}, t) = \frac{i}{2} \vec{n} \times \vec{J} (\vec{r}, t) + \frac{i}{4\pi} \int_{S_0} \nabla \times \vec{J} (\vec{r'}, t) \frac{dS'}{R}$$

(5)

where $S_0$ denotes the surface without singular point. Applying a vector identity [4], the curl term inside the integral of (5) is divided into two terms.

$$H^S (\vec{r}, t) = \frac{i}{2} \vec{n} \times \vec{J} (\vec{r}, t)$$

$$+ \frac{i}{4\pi} \int_{S_0} \left[ \frac{1}{c} \frac{\partial \vec{J} (\vec{r'}, t)}{\partial t} \frac{dS'}{R} + \frac{1}{R} \vec{J} (\vec{r'}, t) \right] \times \frac{dS'}{R^2}.$$

(6)

By substituting (6) into (1) the time-domain magnetic field integral equation (TD-MFIE) results